John Austin

6	7		8	9	5	2	4	1
	4		7	3	1	9	5	6
	1		6	4	2	8	7	3
9				8			3	5
2				6				4
4	3			7				2
1	9	5	4		8		6	
		4	3		6		2	

World of Sudoku

A step-by-step guide to solving Sudoku and Killer Sudoku, as well as other puzzles, with over 190 individual diagrams

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Synopsis

Sudoku and its variants have become an increasing form of entertainment in recent years for a large number of people. They can of course be solved by brute force trial-and-error methods, but with the strategy described here, such methods are rarely needed. This book is a step-by-step guide to that strategy, described in 15 Chapters with almost 200 individual diagrams. Part 1 is devoted to Sudoku itself, and in 8 Chapters it describes the solution method, increasing in complexity from solving the very simple elements and progressing to elements which are linked 5 or more along a row, column or 3x3 square. Part 2 describes Sudoku related puzzles such as Samurai Sudoku and I-doku. Later, the book describes Kakuro and the challenging Killer Sudoku, again providing step-by-step instructions. In Chapter 14, some mathematical background is given and a Sudoku with the minimum number of clues is presented along with its solution.

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1. Introduction

Every decade or so, a new mathematical puzzle seems to enter the public consciousness and become popular. In the 1980s and 1990s it was the turn of Rubik's cube[1] which dismayed millions as they tried desperately using the very limit of their intelligence to rearrange coloured components to form single coloured faces. Solving the cube was distinctly non-trivial, despite appearances to the contrary, and many people must have given up in frustration! Except for a tiny number of people, the solution involved the rote learning of something like 50 twists of the cube from a random orientation. This task is highly suited to young children who can carry out the operations with the least delay. The record for completing the cube currently stands at about 5.5 seconds for a single solve and 6.5 seconds for an average solve[1], an unimaginably short time for those of us with plodding middle-aged brains. The solution of the cube, then, is no longer about intelligence, although the very best are pretty

smart in a geekish, brattish sort of way! My "best time" was about 5 min,delayed as I was by trying to understand the instructions that came with my cube, written as they were in a sort of Chinese English.

Sudoku is an all together different phenomenon. The puzzle is usually in the form of a 9x9 grid composed of 3x3 squares. The task is to complete the 9x9 grid so that each row, column and 3x3 square contains each of the digits 1-9 once and once only. The task is enabled by the grid initially containing some information, usually the numbers present in particular spaces. The 3x3 squares are known as regions and the initial information are sometimes known as 'givens'. However, in this book I will refer to them as 'poles', partly to reflect proper English usage – namely 'given' is an adjective, not a noun! 'Pole' is intended to convey the idea that the other points move around, in some sense, the fixed numbers.

Virtually all newspapers now include a Sudoku puzzle, and it has been credited with the potential to improve mathematics education and to revive, or at least maintain, newspaper sales[2]. However, after almost a decade there seems to be little sign of either! No doubt an exotic sounding name and the fact that it was popular amongst the devilishly clever Japanese has contributed to the success of Sudoku. Indeed, it is now rated by the world

media as the fastest growing puzzle in the world[3]. The great advantage in wasting your time on Sudoku problems is that you need no specialist equipment – just a pen or pencil – and there is no rote learning needed. In most cases, the puzzle can be completed with pure logic alone and in the rest of the cases, only a small amount of trial and error may be needed.

You might think that an ex-mathematician such as myself would have responded positively to these puzzles but it was only early in 2013 that I finally tried to find out what the fuss was all about. Perhaps, like many, I have now become hooked! To be honest, I was put off Sudokus in depth by my dear wife, who claimed that solving them was hard slog – mostly trial and error which doesn't interest me. When I looked at them I simply couldn't see any obvious method, so I tended to agree with my wife, as is generally good for me in any case. In my travels in the early part of 2013, I had chance meetings with several people who had the opposite view. Namely, very little if any trial and error was strictly necessary. It didn't seem important at the time. Nonetheless, the contradiction gnawed away at me until finally I needed to get a clear answer of how to solve Sudoku problems.

This book is the result of a self-imposed study involving the solution of hundreds of Sudoku puzzles over a

period of a few months. Since then, I have solved hundreds more. I have never been particularly concerned with speed: I usually manage them in 20-40 minutes. I know that some newspapers can set ridiculously short targets – perhaps as short as 7 *min*. I assume that I am simply slow-witted compared with the average *Times* reader. Perhaps there is something I have yet to learn. However, my current view is that I have learnt all that I possibly can, and I think this book is a timely distillation of my methods and understanding. It is plausible that other solvers use clever techniques that are beyond my wit and I would be grateful if the reader does not mock the afflicted, namely me!

However you view Sudokus, they do provide a harmless diversion from other activities and provide the mental stimulus that medical practitioners deem is important to (in the words of Agatha Christie's *Hercule Poirot*) 'stimulate the grey cells', and may help to ward off debilitating illnesses such as Alzheimer's disease[4].

As noted above, Sudoku does not require special equipment. If the grid appears in the newspaper, a pen is usually best as newsprint does not hold pencil markings very well. Grids in a book or on better paper are often best completed in pencil. So it is useful to be armed with a rubber for removing the embarrassing mistakes that occur from time to time due to loss of concentration. Another

thing I like to do is to write possible numbers in the top corner of each square. This can help to focus on the essentials. Also, I like to circle my added numbers, especially if I am using a pen, which has the advantage of making the poles stand out from the numbers added. If you then subsequently botch it up, it is easy to transfer the poles to a new grid and start again! In the early months of my Sudoku practice, it was not unusual from time to time, for me to get confused and unable to backtrack my logic to find my errors. Since I don't like anything beating me (!), I usually then draw a new grid and start again from scratch. Typically about 10% of my attempts to solve Sudoku puzzles used to leave me scurrying for scrap paper to redraw the grid! Nowadays, the figure is more like 1%, as my experience has grown.

An issue which I remain confused about is how to solve difficult problems. If iteration (trial and error) is necessary, it suggests that more than one solution might exist. In solving problems, I am always concerned about the 'uniqueness' of solutions. Just because the publisher of the Sudoku book tells you that all the puzzles have only one solution, it doesn't make it true! In fact on many occasions, I have found multiple solutions. The interesting aspect of the systematic methods of Part 1 of this book, is that if these are not sufficient to solve the puzzle, the possibility

of multiple solutions must be considered. As such, I often find 'very difficult' Sudokus tedious rather than stimulating, although perhaps many 'experts' may disagree. Many of the more mathematical ideas are discussed in Part 2. For example, what is the minimum number of poles necessary for a unique solution, and how are Sudokus composed? This may sound a bit academic, and I suppose it is, but sometimes it has practical applications. For example, once I solved a Sudoku which had a low number of poles and I was able to conclude fairly strongly , that there was likely to have been a misprint.

1.1 Historical Background

Sudoku was invented by Howard Garns in 1979 and was initially called 'Number Place'. After being introduced into Japan it was called *Suji wa dokushin ni kagir,* or in English 'The numbers must be single'. The puzzle was later redesigned under the name Sudoku by Maki Koji[3, 5] with the poles placed symmetrically as in a crossword. Originally launched in 1980 in a puzzle magazine, it became very popular in Japan, but it attracted initial attention at first. It was taken up by Wayne Gould of New Zealand and published in the New Hampshire Conway Daily Sun in September 2004. Within a few months it was

enthusiastically taken up by several British newspapers, including the Times, Daily Mail and the Daily Telegraph. The number of regular Sudoku players is now thought to exceed 100 million people worldwide.

A completed Sudoku puzzle is conceptually related to *magic squares* and *Latin squares*. In a magic square, all the rows and columns as well as the long diagonals sum to the same number. Moreover each element of the square is different. One of the simplest is the following:

4	**9**	**2**
3	**5**	**7**
1	**8**	**6**

Here, the rows, columns and diagonals sum to 15, and the square uses just the consecutive numbers 1-9.

Magic squares were known by Chinese mathematicians several thousand years ago dating back to 650 BC or earlier [6]. They can be as large as we wish. A well-known example is the 4x4 square of Albrecht Durer, which contains the double entry 1514, the year in which it was constructed.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Here the sum of the rows is 34, and the composition has the further benefit that regular patterns of subsets of the square also add to be 34 and there are many other features of Durer's composition as described by Bellos[7]. Some of these features are of course accidental, and interestingly, a minor perturbation to the magic square is obtained by turning it upside down and subtracting 1 from the squares with 11, 12, 15 and 16, which gives the following:

1	14	14	4
11	7	6	9
8	10	10	5
13	2	3	15

This appears on the Sagrada Familia cathedral in Barcelona[6,7] and was designed by Antoni Gaudi. The square is not magical, but somehow is considered special.

All the rows, columns and diagonals now add up to 33, thought to be the age of Christ at his death. This writer is amazed, not at the square, but at how anyone can think it's especially significant. The same mystics might equally use the Durer's magical square, which is more fundamental, to "prove" that the history books are wrong and use it to suggest that Christ died at 34. Bellos (*op. cit.*) quotes the number of magic squares as increasing extremely rapidly, faster than the exponential of the grid size. For a 1x1 grid, there is trivially 1 solution, for 2x2 there are none, there are 8 ways for a 3x3, although this includes reflections and rotations. excluding perturbations of the same grid, for a 4x4 grid, the number of possible magic squares is 880, while for a 5x5 grid the number is 275,305,224 [8] and for a 6x6 grid the total is not known exactly, but is an astounding 1.77×10^{19} to a good approximation[8].

A modified form of the magic square is a Latin square in which each row and column contains a symbol only once. This was invented by Leonard Euler in the 18th century [6] and could contain numbers or symbols. The Sudoku is a special case of a Latin square using the numbers 1-9 on a 9x9 grid.

1.2 Book Layout

One of the main purposes of this book, covering Part 1, is to supply a systematic technique to solve Sudoku puzzles, primarily aimed at beginners or those with modest skills. Accordingly, I start in Chapter 2 from first principles and the material becomes gradually more complicated as it progresses through the Chapters. Even experts might benefit from reviewing the material as I am confident there is something for everyone in Chapters 3 to 7. In Chapter 8 I explore the issue of very difficult problems. Such examples appear regularly in newspapers such as *The Times* or *Sunday Times*, and there is a systematic method but some trial and error may be necessary. This complements the earlier Chapters where the solutions are based on logic alone. Chapter 8 also includes one puzzle containing the minimum number of poles (or clues) as a challenge to the reader.

Advanced material is included in Part 2. This includes Sudoku-related problems Gefbadchi (Chapter 9), Sudoku on non-standard grids, often called mini-Sudoku or Super Sudoku (Chapter 10). In Chapter 11, modified Sudoku puzzles, I-doku and Samurai Sudoku (Chapter 12) are presented, with step-by-step solutions for the former.

Chapter 12 covers Kakuro in detail and and Chapter 13 step-by-step solutions are shown for the mother of all Sudoku puzzles, Killer Sudoku! Chapter 14 provides some mathematical discussion of Sudokus, including discussion of the total number of possible Sudokus and the minimum number of poles for a unique solution. Uniqueness is a strict requirement of a Sudoku puzzle, but is not always realised in practice. By way of a concluding section, Chapter 15 suggests sources of Sudoku puzzles for solving and alternative methods of solving other than using your own brain power. The latter is strictly for the lazy or desperate!

Finally, a note on the display of material within this ebook. Sudoku puzzles are built into the ebook code so that comments on each stage in the solution process can be viewed above or below the puzzle. By clicking on the word 'top' which appears above most of the partially completed puzzles, that part of the text will move to the top of the ebook page. This should ensure that no puzzle extends onto a second page when viewed on an ebook reader. Also, the material is best viewed with a type size less than 7, above which the grid may again extend beyond a single ebook page. However, in the case of some of the extended puzzles such as Samurai Sudoku or Kakuro on a large grid, viewing with a small type size will be necessary.

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Biography of the Author

The Author, Dr. John Austin, has over 30 years' research experience on the upper atmosphere and has published almost 90 papers in numerous international scientific journals. In addition John worked for 4 years as an Editor of the Journal of Geophysical Research, the premier Geophysics journal in the USA.

He has spent several years working in the USA, at NASA Langley, Hampton, Virginia (1984-1985) and the University of Washington (1988-1990), where amongst other things he met his future wife Alda, to whom he is still married. During 2003-2011 John worked in Princeton, NJ, USA. His main scientific contribution has been to show the connection between ozone depletion and climate change. John has been involved in the writing of numerous international reports for the World Meteorological Organisation and The Intergovernmental Panel on Climate Change, for which the IPCC received the 2007 Nobel peace prize.

In recent years, John has broadened his work into popular science, through the website http://www.DecodedScience.com and in 2014 he created an internet scientific publishing business Enigma Scientific

Publishing, http://www.enigmascientific.com. "Measuring the World" was his first popular science book (available as an ebook on Amazon), describing practical every-day uses of the metric system.

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When not working, John enjoys a variety of activities including chess, running, photography and travel. He also spends an unhealthy amount of time on mathematical puzzles and enjoying red wine!